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AUTHOR Daniel, Mark
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ABSTRACT

A battery of aptitude and vocabulary tests was administered to 477 students from two high schools. Test scores were correlated with grades in three mathematics courses (algebra, geometry, and algebra II/trigonometry) and scores on the PSAT and SAT. It was found that course grades are most strongly related to analytical reasoning, English vocabulary, and several tests of short-term memory. Scores on the mathematics sections of the PSAT and SAT are predicted by the same aptitudes, plus structural visualization. The verbal sections of the PSAT and SAT are largely predicted by vocabulary, but analytical reasoning, ideaphoria and silograms (verbal short-term memory) also contribute to prediction. (Author)

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RELATIONSHIP OF APTITUDES TO PERFORMANCE IN HIGH SCHOOL MATHEMATICS

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Mark Daniel

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JOHNSON O'CONNOR RESEARCH FOUNDATION
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Technical Report 1982-2

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Relationship of Aptitudes to Performance in High School Mathematics

Mark Daniel

Summary: A battery of aptitude and vocabulary tests was administered to 477 students from two high schools. Test scores were correlated with grades in three mathematics courses (algebra, geometry, and algebra II/ trigonometry) and scores on the PSAT and SAT. It was found that course grades are most strongly related to analytical reasoning, English vocabulary, and several tests of short-term memory. Scores on the math sections of the PSAT and SAT are predicted by the same aptitudes, plus structural visualization. The verbal sections of the PSAT and SAT are largely predicted by vocabulary, but analytical reasoning, ideaphoria and silograms (verbal short-term memory) also contribute to prediction.

Over the past several years the Johnson O'Connor Research Foundation has had the opportunity to administer its battery of aptitude and vocabulary tests to a large number of students at two New York City area high schools. This testing, as well as similar testing at other schools and colleges around the country, has been made possible by a generous grant from a private foundation. The purpose of the grant support has been to provide aptitude testing free of charge to students who can use the information in planning their careers. At the same time, the schools provide to the O'Connor Foundation information on the students' course grades and scores on college entrance examinations, to be used in validating the aptitude tests. The present report is an analysis of the relationship between aptitudes and performance in mathematics courses and on the Preliminary Scholastic Aptitude Test (PSAT) and Scholastic Aptitude Test (SAT) of the College Entrance Examination Board.

The aptitude tests used in this study have been developed over a period of sixty years by the Johnson O'Connor Research Foundation. They measure a wide range of aptitudes in the areas of spatial ability, reasoning, memory, dexterity, musical ability, perceptual speed and idea fluency. In addition, the test battery includes a measure of English vocabulary, which is considered to be learned knowledge rather than an unlearned aptitude. Table 1 lists and describes the aptitudes measured. The tests have reasonably high reliability (the median internal-consistency reliability coefficient is .87), and they have been utilized in numerous occupational and educational validation studies.

Performance in mathematics courses has often been found to be related to the aptitudes of structural visualization and graphoria (clerical speed), and to verbal ability or vocabulary. Structural visualization correlates with grades in a variety of math courses (Bennett, Seashore and Wesman, 1974; McGee, 1979). However, an O'Connor Foundation study (Rippere, 1943) found the Wiggly Block test to have only low correlations with grades in geometry (.11) and algebra (.01). Mathematics in general requires the ability to visualize patterns and relationships between numbers or objects,

Table 1
Aptitudes Measured

Name	Description
Graphoria	Number-checking or clerical speed; speed of comparing pairs of numbers to see whether they are the same or different.
Ideaphoria	Verbal fluency, rate of flow of ideas.
Foresight	Keeping one's mind on a distant goal, taking a far-signted viewpoint.
Inductive Reasoning	Quickness in seeing a relationship among separate facts, ideas or observations.
Analytical Reasoning	Arranging ideas into a logical sequence.
Structural Visualization	Visualizing an object in three dimensions, imagining how a solid object will look when taken apart or viewed from different angles. Measured by the Wiggly Block test.
Tonal Memory	Remembering a sequence of tones.
Pitch Discrimination	Distinguishing between tones very close to each other in pitch.
Rhythm Memory	Remembering a complex rhythm pattern.
Personality	When dealing with people, tending to react from a general, objective viewpoint (high raw score) vs. reacting from a personal, subjective viewpoint (low raw score).
Memory for Design	Memory for straight-line patterns.
Silograms	Remembering paired nonsense and English words.
Number Memory	Remembering several six-digit numbers simultaneously.
Observation	Memory for fine visual details.
Finger Dexterity	Speed and accuracy in manipulating small objects with the fingertips.
Tweeaer Dexterity	Speed and accuracy in handling small objects with tweezers.
Grip	Strength of grip.
English Vocabulary	Knowledge of general English vocabulary.

and so it might be expected that spatial ability would facilitate mathematics performance. The relationship of verbal ability or vocabulary to mathematics grades, which is stronger than the relationship of spatial ability to math grades (Bennett et al., 1974; Human Engineering Laboratory, 1939), may be attributed to the pervasive role of verbal comprehension and expression in all academic areas. Graphoria, which is speed and accuracy in number-checking, has a low to moderate correlation with math grades: Bennett et al. (1974) report correlations of approximately .1, whereas Rippere (1943) found values of approximately .4. A useful overview of the relationship of several ability areas to high-school math performance is provided in the manual for the Differential Aptitude Tests (Bennett et al., 1974). The verbal reasoning and abstract reasoning (figure series) subtests of the DAT correlate about .3 with algebra and geometry grades, the space relations subtest correlates about .2 with grades, and the clerical speed subtest correlates about .1. There is no noticeable difference between the correlations with algebra grades and those with geometry grades.

Although the research just cited, and similar studies, have suggested that verbal ability, spatial ability and clerical speed have some relationship to mathematics performance, several other aptitudes have not been as thoroughly investigated. In particular, there is little previous research correlating various types of memory, ideational fluency, and reasoning tests with mathematics grades. The present study will provide some information about these aptitudes.

Samples and Procedures

Two high schools participated in this study. School 1 is a public school oriented toward communication arts, and School 2 is a Catholic girls' school. Test and grade data were available for a total of 342 students from School 1, ranging in age from 13 to 19. However, because aptitude test performance increases with age during the teenage years, and because 90% of the students tested from School 1 are between the ages of 15 and 17, it was decided to restrict this sample to just the students in the narrower age range. Of these 309 students, 204 (66%) are females. School 2 provided grade data on 168 students, all females, who had taken at least some of the aptitude tests.

Grades were available for several different mathematics courses at each school. In most instances, the grade data were obtained from the schools after the students had taken the aptitude tests. Only those courses were analyzed for which at least fifty cases were available; these courses are algebra, geometry, and algebra II/trigonometry. In addition, School 2 provided overall grade point averages for all subjects for three school years. The schools also provided PSAT and SAT scores. Descriptive statistics on age, course grades and college entrance examination scores are given in Table 2.

The students took the standard battery of aptitude and knowledge tests offered by the Johnson O'Connor Research Foundation. The battery includes both individually-administered and group-administered tests, and takes one-and-a-half to two days to complete (including discussion of

Table 2

Means and Standard Deviations of All Variables

Variable	School 1 (Ages 15-17 only) N	Mean	SD	School 2 N	Mean	SD
Age (at O'Connor testing)	309	15.85	.77	168	17.01	.42
Algebra	181	1.84[a]	.81	164	85.2	9.8
Geometry	285	2.01	.82	162	81.9	10.1
Algebra II/Trigonometry	208	1.89	.89	113	81.7	11.0
Overall grade point avg.	---	-----	----	167	84.5	6.3
PSAT-Verbal	202	439	103	162	415	78
PSAT-Math	200	437	103	162	422	87
SAT-Verbal	260	456	109	153	441	85
SAT-Math	257	458	105	153	451	92
Graphoria 613	299	154.5	24.4	156	163.6	19.8
Ideaphoria 161	309	265.5	62.1	130	279.2	50.1
Foresight 307--transformed[b]	308	4.071	.350	163	3.932	.356
raw scores	308	52.2	22.2	163	44.5	21.8
Inductive Reasoning 164	308	138.7	21.2	166	139.8	19.0
Analytical Reasoning 246	308	32.36	7.14	162	31.78	6.56
Wiggly Block AC--trans.[c]	309	.595	.237	166	.487	.320
raw scores[d]	309	2.92	1.49	166	3.05	1.40
Personality 35	309	17.57	6.64	164	18.61	6.05
Tonal Memory 498	305	48.2	18.8	122	45.8	14.5
Pitch Discrimination 315	309	57.0	11.6	137	53.1	9.5
Rhythm Memory 366--trans.[e]	307	.861	.166	---	-----	----
raw scores	307	45.18	5.85	111	44.68	5.28
Memory for Design 294	309	67.1	27.0	126	73.4	24.9
Silograms 376	309	39.4	15.4	140	44.8	12.7
Number Memory 165	308	71.6	26.0	111	83.1	25.9
Observation 206	309	65.6	10.5	167	68.7	11.1
Finger Dexterity 16	309	81.7	11.6	124	87.5	9.7
Tweezer Dexterity 18	308	34.0	16.0	165	40.3	18.4
Grip 185	300	103.0	33.4	163	93.5	17.6
English Vocabulary, General Scale Score	309	103.1	20.0	120	104.3	16.0

[a] Grading scale: 3=excellent, 2=good, 1=satisfactory, 0=no credit.

[b] Transformed score = natural logarithm of (raw score + 10).

[c] Raw scores above 5.00 minutes were first converted to 5.00. Then, transformed score = $3 - \log_{10}$(raw score).

[d] Raw scores above 5.00 minutes converted to 5.00.

[e] Transformed score = $2 - \log_{10}(60 - \text{raw score})$. The raw score distribution at School 2 was not extremely skewed, and so the transformation was not applied at School 2.

the test results). The students from School 1 were tested between March, 1978 and January, 1980, and the students from School 2 took the tests between October, 1979 and May, 1980. The students came to the Foundation's office individually or in small groups, and throughout the testing program they were treated in the same way as other people who seek the Foundation's services.

Table 2 presents descriptive data on the aptitude-test scores. Finger Dexterity and Grip are administered to both hands, and in the absence of information about the students' preferred hand, the higher of the right-hand and left-hand scores for each test was used for analysis. On three tests the raw score distributions are highly skewed, and in order to prevent the skew from affecting correlations, logarithmic transformations were applied which produce approximately symmetrical distributions. These transformations are described in Table 2. The Wiggly Block raw scores, which are elapsed times, were also inverted so that high raw scores would indicate good test performance.

Methods of Analysis

The basic purpose of this research is to see how aptitudes are related to mathematics performance. Aptitude test scores will be correlated with the various performance criteria: course grades, grade point average, and PSAT and SAT scores. Because there are age and sex differences on most of the aptitude tests and some of the criteria, the partial correlation technique will be used, in which age and (at School 1) sex are statistically controlled.

As previously mentioned, English vocabulary is thought to be more a reflection of acquired knowledge than of aptitude. The vocabulary test correlates moderately with several of the aptitude tests, particularly analytical reasoning and silograms for which the correlations are between .4 and .5 (Bowker, 1981). Vocabulary is known to be a relatively good predictor of school grades and SAT scores (Bowker, 1981). It will be of some interest, then, to examine the correlations between the aptitude tests and the performance criteria when vocabulary is statistically controlled through partial correlation. If this were not done, we would have a difficult time interpreting the correlation between analytical reasoning (for example) and course grades, because we would not know to what extent this correlation is caused by the common relationship of these variables to vocabulary.

Results

Tables 3-10 present partial correlations between the aptitude tests and the various performance criteria. The first set of correlations in each table have age and sex statistically controlled; in the second set of correlations, age, sex and vocabulary are controlled. Table 11 shows the correlations of age and sex with all of the other variables involved in the study. This table indicates the presence and magnitude of sex differences in the course grades and test scores.

Table 3
Partial Correlations with Grades in Algebra

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	173	153	.31	.16	173	116	.29	.20
Ideaphoria	181	129	.14	.03	181	113	.11	-.01
Foresight	181	160	-.14	-.04	181	119	-.17	-.10
Inductive Reas.	181	163	.07	.06	181	119	.03	.03
Analytical Reas.	180	159	.22	.33	180	119	.16	.25
Wiggly Block	181	163	.08	.13	181	119	.09	.11
Personality	181	161	.22	.01	181	119	.21	-.02
Tonal Memory	177	121	.12	.12	177	116	.08	.03
Pitch Disc.	181	136	.08	.03	181	115	.05	.00
Rhythm Memory	179	110	.16	.11	179	109	.13	.07
Memory for Design	181	125	.28	.04	181	116	.26	.05
Silograms	181	139	.34	.17	181	114	.29	.06
Number Memory	180	110	.30	.19	180	109	.30	.12
Observation	181	164	.18	-.07	181	119	.20	-.07
Finger Dexterity	181	122	.12	.04	181	116	.12	.08
Tweezer Dexterity	180	162	.06	.00	180	119	.07	.02
Grip	175	160	-.18	-.06	175	119	-.18	-.06
Vocabulary	181	119	.20	.35	---	---	--	--

Table 4
Partial Correlations with Grades in Geometry

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	277	152	.27	-.02	277	116	.25	.00
Ideaphoria	285	128	.26	.04	285	113	.23	.00
Foresight	284	158	-.04	.10	284	118	-.07	.06
Inductive Reas.	284	161	.10	-.03	284	118	.07	-.05
Analytical Reas.	284	157	.35	.35	284	118	.30	.29
Wiggly Block	285	161	.14	.12	285	118	.15	.11
Personality	285	159	.13	.03	285	118	.12	.00
Tonal Memory	282	120	.13	.06	282	116	.10	-.02
Pitch Disc.	285	135	.15	.09	285	115	.12	.07
Rhythm Memory	284	109	.18	.11	284	109	.15	.08
Memory for Design	285	124	.23	.15	285	116	.21	.17
Silograms	285	138	.23	.27	285	114	.17	.20
Number Memory	284	109	.21	.30	284	109	.20	.25
Observation	285	162	.08	-.01	285	118	.10	.00
Finger Dexterity	284	121	.13	.01	284	116	.13	.04
Tweezer Dexterity	284	160	.14	.10	284	118	.15	.11
Grip	276	158	-.15	-.07	276	118	-.15	-.08
Vocabulary	285	118	.20	.27	---	---	--	--

Table 5

Partial Correlations with Grades in Algebra II/Trigonometry

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	202	105	.25	.01	202	87	.23	.03
Ideaphoria	208	91	.20	.11	208	87	.16	.08
Foresight	208	111	-.02	.06	208	87	-.05	.01
Inductive Reas.	207	113	.04	.03	207	87	-.01	.01
Analytical Reas.	208	110	.17	.30	208	87	.09	.22
Wiggly Block	208	112	.14	.21	208	87	.14	.20
Personality	208	112	.12	.04	208	87	.11	.01
Tonal Memory	205	89	.03	.11	205	87	-.01	.03
Pitch Disc.	208	94	-.05	.04	208	87	-.09	.01
Rhythm Memory	208	78	.08	.20	208	78	.04	.18
Memory for Design	208	90	.13	.18	208	87	.15	.19
Silograms	208	93	.29	.21	208	87	.23	.12
Number Memory	207	78	11	.28	207	78	.10	.23
Observation	208	113	.08	.00	208	87	.10	.01
Finger Dexterity	208	89	.11	.00	208	87	.12	.04
Tweezer Dexterity	207	111	.00	.11	207	87	.00	.13
Grip	204	111	-.10	-.05	204	87	-.10	-.05
Vocabulary	208	87	.23	.30	---	---	--	--

Table 6

Partial Correlations with Overall Grade Point Average (School 2 Only)

Test Name	Controlling for Age		Controlling for Age and Vocabulary	
	N	Corr.	*N*	Corr.
Graphoria	157	.09	116	.16
Ideaphoria	131	.16	113	.09
Foresight	163	.14	121	.06
Inductive Reas.	166	.05	120	.00
Analytical Reas.	162	.42	121	.31
Wiggly Block	166	.09	121	.06
Personality	164	.09	121	.03
Tonal Memory	123	.12	116	-.05
Pitch Disc.	138	.10	115	.06
Rhythm Memory	112	.12	109	.07
Memory for Design	127	.08	116	.12
Silograms	141	.36	114	.22
Number Memory	112	.30	109	.21
Observation	167	.00	121	.01
Finger Dexterity	125	-.02	116	.04
Tweezer Dexterity	165	.12	121	.17
Grip	163	-.03	121	-.04
Vocabulary	121	.57	---	--

Table 7
Partial Correlations with PSAT-Verbal

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	197	152	.13	.05	197	116	.07	.14
Ideaphoria	202	129	.27	.26	202	113	.18	.23
Foresight	201	158	.07	.04	201	120	-.05	-.09
Inductive Reas.	201	161	.18	.15	201	120	.06	.13
Analytical Reas.	201	157	.55	.33	201	120	.41	.16
Wiggly Block	202	161	.07	.09	202	120	.12	.05
Personality	202	159	.23	.15	202	120	.26	.11
Tonal Memory	199	122	.20	.22	199	116	.08	.03
Pitch Disc.	202	136	.22	.03	202	115	.14	-.05
Rhythm Memory	202	111	.21	.13	202	109	.12	.06
Memory for Design	202	126	.17	.03	202	116	.08	.08
Silograms	202	138	.36	.35	202	114	.13	.16
Number Memory	201	111	.05	.23	201	109	.02	.10
Observation	202	162	.05	.05	202	120	.16	.09
Finger Dexterity	202	124	-.11	.00	202	116	-.13	.10
Tweezer Dext.	201	161	-.02	-.08	201	120	.01	-.07
Grip	196	158	-.06	-.06	196	120	-.06	-.08
Vocabulary	202	120	.72	.70	---	---	--	--

Table 8
Partial Correlations with PSAT-Math

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	195	152	.18	.16	195	116	.15	.21
Ideaphoria	200	129	.35	.15	200	113	.31	.10
Foresight	199	158	.07	.10	199	120	.02	.04
Inductive Reas.	199	161	.11	.13	199	120	.05	.10
Analytical Reas.	199	157	.50	.41	199	120	.43	.31
Wiggly Block	200	161	.32	.23	200	120	.34	.23
Personality	200	159	.26	.07	200	120	.26	.03
Tonal Memory	197	122	.20	.14	197	116	.15	.02
Pitch Disc.	200	136	.26	.15	200	115	.22	.12
Rhythm Memory	200	111	.21	.24	200	109	.17	.21
Memory for Design	200	126	.24	.21	200	116	.20	.25
Silograms	200	138	.25	.23	200	114	.14	.09
Number Memory	199	111	.24	.26	199	109	.24	.17
Observation	200	162	.12	.07	200	120	.16	.09
Finger Dexterity	200	124	.11	.16	200	116	.14	.23
Tweezer Dexterity	199	161	.06	.07	199	120	.08	.10
Grip	194	158	.08	.01	194	120	.13	.01
Vocabulary	200	120	.32	.45	---	---	--	--

Table 9
Partial Correlations with SAT-Verbal

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	251	143	.22	.12	251	110	.19	.23
Ideaphoria	260	121	.28	.22	255	110	.20	.17
Foresight	259	149	.00	-.03	254	110	-.15	-.19
Inductive Reas.	259	152	.23	.13	254	110	.13	.10
Analytical Reas.	259	148	.48	.36	254	110	.29	.20
Wiggly Block	260	152	-.02	.04	255	110	-.01	-.01
Personality	260	150	.15	.20	255	110	.15	.18
Tonal Memory	256	115	.18	.11	251	110	.05	-.12
Pitch Disc.	260	127	.17	.01	255	110	.06	-.07
Rhythm Memory	260	104	.18	.10	255	104	.08	.03
Memory for Design	260	118	.18	.05	255	110	.10	.09
Silograms	260	128	.42	.27	255	110	.22	.05
Number Memory	259	104	.13	.21	254	104	.14	.08
Observation	260	153	.02	.09	255	110	.11	.13
Finger Dexterity	260	116	.00	.03	260	110	.14	.03
Tweezer Dexterity	259	151	.04	-.13	254	110	.09	-.13
Grip	253	149	-.08	-.03	253	110	-.09	-.03
Vocabulary	260	110	.72	.69	---	---	--	--

Table 10
Partial Correlations with SAT-Math

Test Name	Controlling for Age and Sex				Controlling for Age, Sex and Vocabulary			
	N		Corr.		*N*		Corr.	
	1	2	1	2	1	2	1	2
Graphoria	248	143	.19	.09	248	110	.15	.14
Ideaphoria	257	121	.26	.19	257	110	.20	.15
Foresight	256	149	.01	.17	256	110	-.05	.11
Inductive Reas.	256	152	.09	.14	256	110	.01	.12
Analytical Reas.	256	148	.49	.48	256	110	.40	.39
Wiggly Block	257	152	.23	.28	257	110	.25	.28
Personality	257	150	.23	.12	257	110	.22	.09
Tonal Memory	253	115	.21	.13	253	110	.15	.01
Pitch Disc.	257	127	.23	.18	257	110	.18	.15
Rhythm Memory	257	104	.18	.21	257	104	.13	.18
Memory for Design	257	118	.34	.25	257	110	.31	.29
Silograms	257	128	.30	.26	257	110	.18	.12
Number Memory	256	104	.31	.31	256	104	.31	.23
Observation	257	153	.14	.16	257	110	.18	.19
Finger Dexterity	257	116	.15	.07	257	110	.17	.13
Tweezer Dexterity	256	151	.11	-.06	256	110	.13	-.04
Grip	250	149	-.03	.00	250	110	-.03	.00
Vocabulary	257	110	.38	.46	---	---	--	--

Table 11
Correlations of Age and Sex with Course Grades and Test Scores

Variable	Age		Sex[a]
	School 1	School 2	(School 1)
PSAT-Verbal	-.08	-.03	-.11
PSAT-Math	-.14	-.21	-.22
SAT-Verbal	-.13	-.02	-.10
SAT-Math	-.04	-.17	-.26
Algebra	-.08	-.15	.13
Geometry	-.03	-.14	.01
AlgII/Trig.	.00	-.11	.01
Overall GPA	--	-.11	--
Graphoria	.02	-.07	.08
Ideaphoria	.03	-.11	.14
Foresight	.09	-.11	.10
Inductive Reas.	.01	-.11	.17
Analytical Reas.	-.18	-.06	.00
Wiggly Block	.03	.02	-.27
Personality	-.11	.01	-.10
Tonal Memory	.04	.05	.02
Pitch Disc.	-.03	-.06	-.13
Rhythm Memory	.10	-.08	-.01
Memory for Design	.07	-.22	.02
Silograms	-.05	-.07	.19
Number Memory	.01	-.08	.11
Observation	-.04	.13	.10
Finger Dexterity	.14	.03	.31
Tweezer Dexterity	.08	-.05	.14
Grip	.15	-.10	-.71
Vocabulary	.04	-.06	.01

Note: For sample sizes, see Table 2.
[a] Male=1, female=2; therefore a positive correlation indicates that females score higher than males, and a negative correlation shows that males score higher than females.

Course grades. The patterns of correlations with course grades tend to be different in the two schools, but tend to be similar across the three different subject areas within each school. Looking first at the partial correlations controlling for age and sex, but not for vocabulary, at School 1 the silograms and graphoria aptitude tests are among the strongest predictors of grades in the three courses. The graphoria correlations range from .25 to .31, indicating that about eight per cent of the variance in individual course grades can be explained by a simple number-checking test. Silograms measures short-term memory for nonsense words, and its preeminent position as a predictor in two of the three subjects might be explained by the fact that it measures in part a general memory factor and also correlates about .4 with vocabulary, which is a relatively good predictor of academic performance. This interpretation is strengthened by the fact that two other short-term memory tests--number memory and memory for design--are only slightly less predictive than silograms in two of the three courses. The analytical reasoning test is the best single predictor of geometry grades, and it correlates moderately (.17,.22) with grades in the other two courses; however, like silograms it correlates with vocabulary, and its predictive validity may be due in part to this overlap with verbal ability. Vocabulary itself correlates only moderately (between .20 and .23) with grades in these three subjects.

In contrast, at School 2 analytical reasoning and vocabulary are the outstanding predictors of performance in all three courses. The analytical reasoning correlations range from .30 to .35, and the vocabulary correlations from .27 to .35. As at School 1, two memory tests (silograms and number memory) have relatively high correlations with grades in two of the three subjects. Unlike School 1, at School 2 graphoria does not correlate substantially with course grades.

It might be suspected that the difference between the results from the two schools is related to the difference in the sex composition in the two samples. To investigate this, the correlations at School 1 were recalculated separately for males and females, and it was found that the correlations for each sex were approximately the same as the overall correlations shown in Table 3-5. Therefore, the difference in results cannot be explained by sex differences.

Overall, when age and sex are statistically controlled, analytical reasoning, silograms, number memory and English vocabulary have moderate to relatively high correlations with course grades in both schools. Graphoria correlates substantially with grades at School 1, but has low correlations at School 2. Surprisingly, Wiggly Block, the measure of structural visualization, is not a notable predictor of grades in any of the three courses; its average correlation is .13.

When vocabulary is statistically controlled, along with age and sex, the results do not change greatly. The magnitudes of the correlations tend to shrink, especially for silograms and analytical reasoning, which are most strongly correlated with vocabulary; but for several tests which have low correlations with vocabulary, including graphoria, memory for design and number memory, there is little if any reduction in the size of the correlations. Analytical reasoning is still the best single predictor of grades

in geometry at both schools, and although it is a good predictor of grades in algebra and algebra II/trigonometry at School 2, at School 1 these correlations become modest (.09,.16). The three memory tests--silograms, number memory and memory for design--are approximately equal in their predictive power after vocabulary is controlled. This result is somewhat surprising, because number memory has obvious applications in mathematics in general, and memory for design is clearly useful in geometry, but silograms has no apparent relationship to the content of mathematics. Graphoria still correlates relatively highly with course grades at School 1 after partialling out vocabulary; at School 2 the correlations are slightly higher than they were without the vocabulary control, but they are still near zero for two of the three courses.

The overall grade point average at School 2, including both mathematics and other courses, is quite substantially correlated with vocabulary ($r = .57$). Analytical reasoning is the next best predictor, both before and after controlling statistically for vocabulary; about ten per cent of the non-vocabulary-related variance in GPA can be explained by the analytical reasoning test alone. Silograms and number memory are moderately related to GPA, reinforcing the idea that short-term memory ability is useful in academic work. Finally, graphoria is a modest predictor of school grades, accounting for about 2½ per cent of GPA variance after vocabulary is controlled.

PSAT/SAT Scores. When age and sex are controlled, PSAT-Verbal is best predicted by the same four tests at the two schools: vocabulary (correlations of .72 and .70), analytical reasoning (.55,.33), silograms (.36,.35) and ideaphoria (.27,.26). Virtually the same results are obtained for the SAT-Verbal test: vocabulary (.72,.69), analytical reasoning (.48,.36), silograms (.42,.27) and ideaphoria (.28,.22) are the best predictors.

That vocabulary should be the major predictor of performance on the verbal portions of the PSAT and SAT is not surprising; in fact, previous studies (Bowker, 1977) have found the correlation between vocabulary and SAT-Verbal scores to be even higher, in the range .81-.85. The PSAT-Verbal and SAT-Verbal tests appear to be, to a large extent, measures of vocabulary knowledge. Analytical reasoning, silograms and ideaphoria all correlate moderately with vocabulary, and so their correlations with PSAT-Verbal and SAT-Verbal are in part reflections of a common vocabulary element.

When vocabulary is statistically controlled, the correlations with these three tests (analytical reasoning, ideaphoria and silograms) are substantially reduced. Their mean correlations, with vocabulary controlled, are: analytical reasoning, .29; ideaphoria, .19; and silograms, .16. Thus, analytical reasoning alone predicts about eight per cent of the variance in PSAT-Verbal and SAT-Verbal scores that remains after the influence of vocabulary is removed, ideaphoria predicts about four per cent, and silograms about three per cent. It is noteworthy that previous research (Bowker, 1977) found a correlation of similar magnitude between analytical reasoning and SAT-Verbal. The analytical reasoning test appears to be measuring some ability (presumably a reasoning aptitude) that is distinct from vocabulary and that has a considerable relationship with performance on the PSAT-Verbal and SAT-Verbal.

The aptitude-test correlations with PSAT-Math and SAT-Math are also similar at the two schools, when age and sex are controlled. For the PSAT-Math test, the highest correlations are with analytical reasoning (.50 at School 1, .41 at School 2), vocabulary (.32,.45), Wiggly Block (.32,.23), number memory (.24,.26), silograms (.25,.23) and memory for design (.24,.21). The same tests correlate highly with SAT-Math: analytical reasoning (.49,.48), vocabulary (.38,.46), number memory (.31,.31), memory for design (.34,.25), silograms (.30,.26) and Wiggly Block (.23,.28). In summary, the mathematics portions of the PSAT and SAT are predicted by analytical reasoning, vocabulary, structural visualization, and short-term memory tests. The vocabulary test correlates much less with PSAT-Math and SAT-Math than with PSAT-Verbal and SAT-Verbal; conversely, the Wiggly Block test is more strongly related to the mathematics tests than to the verbal tests. Analytical reasoning is about equally correlated with the math and verbal tests.

Controlling for vocabulary through partial correlation has the effect of greatly reducing the correlations of silograms with PSAT-Math and SAT-Math. The other correlations are reduced only slightly, or, in some cases, are increased. With vocabulary controlled, analytical reasoning has a mean correlation of .39 with the math tests, Wiggly Block correlates .28, memory for design correlates .27 and number memory correlates .26. As with the verbal tests, analytical reasoning predicts a substantial proportion (15%) of the variance in PSAT-Math and SAT-Math scores, beyond that which is predicted by vocabulary. Structural visualization and the two memory tests also are fairly strongly related to math scores.

Discussion

The analytical reasoning test is among the best predictors of all of the academic criteria analyzed in this research: mathematics course grades, overall grade point average, and scores on the verbal and math portions of the PSAT and SAT. This is true even when the influence of English vocabulary knowledge is statistically controlled. The observed correlations with analytical reasoning are all the more impressive when it is realized that this test has quite low internal-consistency reliability (.65; Daniel and Kling, 1974). The analytical reasoning test measures an aptitude for arranging ideas into a logical pattern. It seems reasonable to conclude that this type of reasoning is an important determinant of performance in a range of academic areas.

The analytical reasoning test has previously been analyzed as part of a factor analysis involving marker tests of reasoning and spatial ability, in addition to the standard O'Connor Foundation battery (Trafton, 1981). The test was found to be most strongly related to a factor defined by tests of "logical" or syllogistic reasoning, such as the Diagramming Relationships and Inference tests of the ETS Kit of Factor-Referenced Cognitive Tests. However, it also loads substantially on a spatial-visualization factor defined by Wiggly Block, Paper Folding, Surface Development and Form Board. It is unlikely that the analytical reasoning test's predictive power in the present study is caused to any significant extent by its overlap with spatial ability, because the Wiggly Block test has negligible correlations with most of the criterion variables.

This study has found that tests of short-term memory for three different types of content--nonsense words, numbers, and patterns--make a considerable contribution to the prediction of mathematics course grades. Furthermore, two of these tests, silograms and number memory, are relatively good predictors of overall grade point average; and number memory and memory for design are each relatively highly correlated with scores on the math sections of the PSAT and SAT. It appears, then, that these short-term memory aptitudes, particularly number memory and memory for design, are helpful in learning and/or performing mathematics. It is easy to imagine ways in which short-term memory ability would help a student in a course, such as in mastering new material or studying for a test. The memory tests are not very good predictors of performance on the verbal portions of the PSAT and SAT, after vocabulary is statistically controlled.

Graphoria, or number-checking speed, correlates more highly with mathematics course grades at School 1 than at School 2, for unknown reasons. Perhaps the schools use different teaching or testing methods in these courses. Overall, graphoria accounts for a modest proportion (two or three per cent) of the variance in the grades and test scores, after vocabulary is controlled.

Table 11 shows that females perform at least as well as males in the mathematics courses, but males obtain higher scores on the math sections of the PSAT and SAT. We have also found that structural visualization, as measured by the Wiggly Block, correlates moderately with PSAT-Math and SAT-Math but only negligibly with mathematics course grades. These findings suggest that performance in high-school mathematics courses is nearly independent of structural visualization, but performance on the math portions of college entrance examinations is moderately affected by spatial ability. It may be that the superior male performance on PSAT-Math and SAT-Math is a function of the higher spatial demands of those tests.

Regarding the relationship between structural visualization and math course grades, the present research is in line with earlier findings by the Foundation of a very small relationship (Rippere, 1943), as opposed to the research reported by McGee (1979) and Bennett et al. (1974). The manual for the DAT reports correlations (for large samples) between the space relations subtest and course grades; the median values are .23 for algebra and .13 for geometry. One explanation for the discrepancy between the Foundation's results and outside findings is that the Foundation uses an apparatus test, the Wiggly Block, which has a fairly low correlation (in the .20s) with vocabulary (Daniel, 1981), whereas the DAT's space relations subtest, a paper-and-pencil test, correlates .63 with the verbal reasoning subtest (Bennett et al., 1974). Because the Wiggly Block test is less influenced by verbal ability, it may give a more accurate indication of the true relationship between structural visualization and performance in mathematics courses.

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